Journal of Applied Mechanics and Technical Physics, Vol. 39, No. 2, 1998

ON THE FORMS OF RELATIONSHIP BETWEEN TWO NONCOAXIAL SECOND-ORDER TENSORS (THE CASE OF PLANE STRAIN OR A PLANE STRESS STATE)

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UDC 539.376

A relationship between noncoaxial tensors of stress and creep strain rate is established for the case of plane strain or a plane stress state. The basis is the experimentally substantiated hypothesis on the existence of a creep surface, which is a set of loading paths in the stress space that, at any time, ensure identical values of the creep intensity for a certain chosen measure and orthogonality of the creep strain rate vector to this surface. The relation obtained completely corresponds to available experimental data for complex loading.

1. Engineering theories of creep are based, as is known, on the assumption of similarity between stress deviators and creep strain rate deviators or creep strain deviators [1]. Namestnikov [2–6] studied the agreement between theories and experimental data. He noted [2–5] that there is a systematic deviation of experimental data from the proportionality of the corresponding deviators. A measure of this deviation is the phase of similarity (Novozhilov's terminology), which is the difference in the angle of form between the stress tensor and the creep strain rate tensor or the strain tensor (recall that, in the deviator plane, the angle of form of the stress tensor determines the direction of the stress intensity vector and the angle of form of the strain rate tensor determines the direction of the creep strain intensity vector).

Further analysis of numerous experimental data assuming coaxiality of the stress tensor and the creep strain rate tensor or the creep strain tensor leads to the following conclusions [7–10].

(1) The phase of similarity of the corresponding deviators is an odd function of just the angle of form of the stress state [10].

(2) In the case of a steady stressed state and simple loading, the creep strain rate vector (or the creep strain vector) does not change direction during deformation from the moment of application of the load up to failure of the specimen. This implies that the phase of similarity of the stress deviator and the creep strain rate deviator (or the strain deviator) remains fixed during deformation of the material for both a fixed stressed state and proportional loading. Moreover, precisely in these loading regimes, the phase of similarity of the stress deviator and the creep strain rate deviator is equal to the phase of similarity of the stress deviator and the creep strain deviator [7, 8].

(3) In complex loading, the phase of similarity of the stress deviator and the creep strain rate deviator undergoes an increment that depends on the increment of the angle of form of the stress state and the sign of this increment and tends to zero with time [9].

The experimentally observed "deflection" of the creep strain rate vector in the direction of rotation of the stress vector in complex loading has been previously explained only by the increment of the phase of similarity of the corresponding deviators [9, 11]. This is not always valid. Indeed, in experiments on extension–compression with simultaneous torsion of specimens under complex loading, the principal axes of the stress tensor change orientation. Naturally, the principal axes of the creep strain rate tensor also change

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences, Novosibirsk 630090. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 39, No. 2, pp. 135–140, March–April, 1998. Original article submitted July 9, 1996.

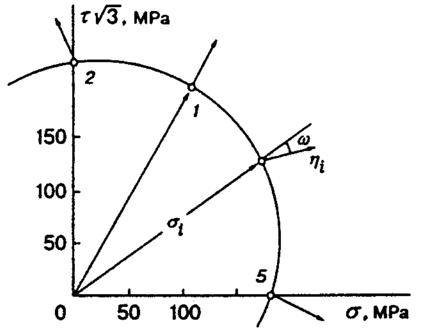

Fig. 1

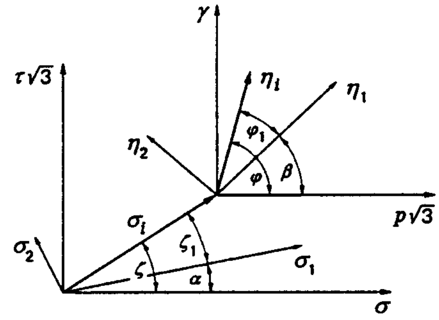

Fig. 2

orientation. It is clear that in the case of complex loading considered, not only does the phase of similarity of the corresponding deviators undergo an increment but also the coaxiality of the stress tensor and the creep strain rate tensor is violated.

To substantiate the last statement, it suffices to analyze the experimental data of [11], obtained on specimens loaded by a tensile load with simultaneous application of a torque (the material was St. 45 and the test temperature was 400°C). The first loading cycle corresponded to a certain combination of tensile and shear stresses (point 1 in Fig. 1).[1] The second loading cycle, carried out by steps, corresponded to either tension (point 5 in Fig. 1), during which shear strain was accumulated simultaneously with axial strain in the specimen material, or pure torsion (point 2 in Fig. 1), during which axial strain was accumulated simultaneously with shear strain in the specimen material. In the experiments of [12] on technically pure copper at 150°C, the first loading cycle corresponded to tension (torsion) of the specimen, and the second corresponded to torsion (tension). From analysis of creep curves it follows that "... after replacement of tension by torsion the elongation strain decreases, and after replacement of torsion by tension the shear strain decreases. The same was observed previously on ÉI-257 steel.... The experimental results agree with none of the existing creep theories" [12]. This should be expected because the experimental facts described above indicate not only disruption of the similarity between the corresponding deviators but also disruption of the coaxiality of the stress tensor and the creep strain rate tensor immediately after the beginning of complex loading.

When the plane $\sqrt{3}p - \gamma$, in which all experimental information on the creep of specimens is presented [9, 11], is superimposed on the plane $\sigma - \sqrt{3}\tau$, in which information on the history of the loading path of these specimens is presented, it becomes evident that the creep strain rate vector is deflected in the direction of rotation of the stress vector [9, 11]. Geometric interpretation of the stress-strain state at any point of the complex loading path is illustrated in Fig. 2 with the same conventional notation as in Fig. 1: σ and τ are the normal and tangential stresses in the cross section of the specimen, σ_i is the stress intensity, σ_1 and σ_2 are the main components of the stress tensor, p and γ are the axial and shear creep strain, η_i is the creep strain rate intensity, η_1 and η_2 are the main components of the creep strain rate tensor, and the remaining designations are obvious. The magnitude of the above-mentioned deflection, as follows from Fig. 2, is evaluated as the difference of the angles $\zeta = \zeta_1 + \alpha$ and $\varphi = \varphi_1 + \beta$, i.e., $\omega = \zeta - \varphi = (\zeta_1 - \varphi_1) + (\alpha - \beta)$.

Obviously, ω is the sum of the departure of the corresponding deviators from proportionality, a measure of which is $\omega_1 = \zeta_1 - \varphi_1$, and the departure of the stress tensor and the creep strain rate tensor from coaxiality, a measure of which is $\omega_2 = \alpha - \beta$. It is clear that for $\omega_2 = 0$, ω is only a measure of departure of the corresponding deviators from proportionality, and for $\omega_1 = 0$, it is only a measure of departure of the stress tensor and the creep strain rate tensor from coaxiality. By analogy with the general case, ω can be called the phase of similarity of the deviators as applied to the plane stress state considered.

[1]Figure 1 shows a fragment of Fig. 2 in [11].

substantiated hypothesis on the existence of a creep surface [7, 9], which is a set of loading paths in the stress space that, at any time, ensure equivalent values of the creep intensity for a chosen measure (hereinafter the value of dissipated energy is used as this measure) and orthogonality of the creep strain rate vector to this surface. The equation of this surface can be generally written as $\sigma_e = \text{const}$.

In the case considered, in the plane $\sigma - \sqrt{3}\tau$, σ_e is a closed convex curve [9]. The equation of this curve is obtained from the above definition of a creep surface. Obviously, the requirement of equivalence of the creep intensity for any loading path reduces to smoothness of the curve of "dissipated work–time":

$$\frac{d(\sqrt{3}p)}{dt}\, d\sigma + \frac{d\gamma}{dt}\, d(\sqrt{3}\tau) = 0. \tag{2.1}$$

Since the loading path belongs to σ_e, equality (2.1) is simultaneously the condition of orthogonality of the creep strain rate vector to σ_e.

From Fig. 2 it follows that $\sigma = \sigma_i \cos\zeta$, $\sqrt{3}\tau = \sigma_i \sin\zeta$, and $\tan\zeta = \sqrt{3}\tau/\sigma$. Taking this into account, from (2.1) we obtain

$$\sigma_i \exp\Big(-\int\limits_{\zeta^0}^{\zeta} \tan\omega\, d\zeta\Big) = \sigma_i^0, \qquad \omega = \zeta - \varphi, \tag{2.2}$$

where σ_i^0 and ζ^0 correspond to the beginning of the loading path. Thus,

$$\sigma_e = \sigma_i \exp\Big(-\int\limits_{\zeta^0}^{\zeta} \tan\omega\, d\zeta\Big). \tag{2.3}$$

Obviously, if at time t, we have $\omega = 0$, the contour of equivalent stress states (2.2) is a Mises classical circle, i.e., $\sigma_i = \sigma_i^0$ and $\sigma_e = \sigma_i$. Taking into account that

$$\frac{d(\sqrt{3}p)}{dt} = \lambda\,\frac{\partial\sigma_e}{\partial\sigma}, \qquad \frac{d\gamma}{dt} = \lambda\,\frac{\partial\sigma_e}{\partial(\sqrt{3}\tau)},$$

after standard operations using (2.3), we write

$$\frac{d(\sqrt{3}p)}{dt} = W\Big[\frac{\sigma}{\sigma_i^2} + \frac{\sqrt{3}\tau}{\sigma_i^2}\tan\omega\Big], \qquad \frac{d\gamma}{dt} = W\Big[\frac{\sqrt{3}\tau}{\sigma_i^2} - \frac{\sigma}{\sigma_i^2}\tan\omega\Big], \tag{2.4}$$

where W is the energy dissipated during the creep process.

Relations (2.4) are final. We analyze the implications of these relations.

(A) In the case of a steady stress state or simple loading, the phase of similarity of deviators, as noted above, remains fixed. Let $\sigma = 0$ and $\tau \neq 0$. From (2.4) it follows that

$$\frac{d\gamma}{dt} = W\,\frac{\sqrt{3}\tau}{\sigma_i^2}, \qquad \frac{d(\sqrt{3}p)}{dt} = W\,\frac{\sqrt{3}\tau}{\sigma_i^2}\tan\omega, \tag{2.5}$$

i.e., in pure torsion, slight axial creep is observed. This was repeatedly detected in the experiments of Trunin [13], including experiments on Ti-6Al-4V titanium alloy at room temperature. Axial creep is absent for $\omega = 0$.

(B) We consider complex loading. We assume that, in the first loading stage, $\sigma \neq 0$ and $\tau = 0$ at $t \leqslant t_1$. Obviously, in uniaxial extension–compression, $\omega = 0$, and from (2.4) we have

$$\frac{d(\sqrt{3}p)}{dt} = W\,\frac{\sigma}{\sigma_i^2}, \qquad \frac{d\gamma}{dt} = 0. \tag{2.6}$$

In the second loading stage, $\sigma = 0$ and $\tau \neq 0$ at $t > t_1$. In complex loading, as noted above, the phase of similarity of the deviators undergoes an increment that tends to zero with time. Thus, in the second loading stage, $\omega \neq 0$. Moreover, from the experiments of Sosnin [11] it follows that $\omega < 0$. From (2.4) we have

$$\frac{d(\sqrt{3}p)}{dt} = W\frac{\sqrt{3}\tau}{\sigma_i^2}\tan\omega, \qquad \frac{d\gamma}{dt} = W\frac{\sqrt{3}\tau}{\sigma_i^2}. \tag{2.7}$$

From (2.6) and (2.7) it follows that, after replacement of extension by torsion, the axial creep strain decreases. Precisely this result was obtained in the experiments of Namestnikov [12].

We now assume that in the first loading stage, $\sigma = 0$ and $\tau \neq 0$. Then from (2.4) we obtain relations (2.5). According to the experiments of [11–13], we set $\omega = 0$ in the first loading stage. Then, we have

$$\frac{d(\sqrt{3}p)}{dt} = 0, \qquad \frac{d\gamma}{dt} = W\frac{\sqrt{3}\tau}{\sigma_i^2}. \tag{2.8}$$

In the second loading stage at $\sigma \neq 0$ and $\tau = 0$, the phase of similarity of the deviators undergoes an increment, i.e., $\omega \neq 0$ and, as follows from [11], $\omega > 0$. From (2.4) we have

$$\frac{d(\sqrt{3}p)}{dt} = W\frac{\sigma}{\sigma_i^2}, \qquad \frac{d\gamma}{dt} = -W\frac{\sigma}{\sigma_i^2}\tan\omega. \tag{2.9}$$

From (2.8) and (2.9) it is obvious that, after replacement of torsion by extension, the shear creep strain decreases. This completely agrees with the experimental results of Namestnikov [12].

(C) We consider an example that corresponds to the experimental data of Sosnin [11]. We assume that in the first loading stage, $\sigma \neq 0$, $\tau \neq 0$, and $\omega = 0$ (point 1 in Fig. 1). Then, we have

$$\frac{d(\sqrt{3}p)}{dt} = W\frac{\sigma}{\sigma_i^2}, \qquad \frac{d\gamma}{dt} = W\frac{\sqrt{3}\tau}{\sigma_i^2}.$$

If in the second stage of complex loading, $\sigma = 0$ and $\tau \neq 0$ (point 2 in Fig. 1) or $\sigma \neq 0$ and $\tau = 0$ (point 5 in Fig. 1), then relation (2.4) immediately leads to relations (2.7) or (2.9), which completely agree with the experimental results of Sosnin [11], presented in Fig. 1. This follows from analysis of the direction of the creep strain rate vector at loading points 2 and 5.

3. The case of plane strain is considered similarly. After standard operations using (2.2), we obtain

$$\dot{p}_x = W\Big[\frac{\sigma_x - \sigma_y}{4\sigma_i^2} + \frac{\tau_{xy}}{2\sigma_i^2}\tan\omega\Big], \quad \dot{p}_y = -W\Big[\frac{\sigma_x - \sigma_y}{4\sigma_i^2} + \frac{\tau_{xy}}{2\sigma_i^2}\tan\omega\Big],$$
$$\dot{p}_{xy} = W\Big[\frac{\tau_{xy}}{\sigma_i^2} - \frac{\sigma_x - \sigma_y}{2\sigma_i^2}\tan\omega\Big] \tag{3.1}$$

(the dot denotes differentiation in time, the remaining notation is conventional, and the expressions for W, σ_i, and ω correspond to the case of plane strain). Relations (3.1) coincide with similar relations obtained by a different line of reasoning in [14]. This suggests the reliability of the method of determining the functional relationship between two noncoaxial second-rank tensors described in the present paper.

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